

# Power-law entropy-corrected HDE and NADE in Brans-Dicke cosmology

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## Abstract

Considering the power-law corrections to the black hole entropy, which appear in dealing with the entanglement of quantum fields inside and outside the horizon, the holographic energy density is modified accordingly. In this paper we study the power-law entropy-corrected holographic dark energy in the framework of Brans-Dicke theory. We investigate the cosmological implications of this model in detail. We also perform the study for the new agegraphic dark energy model and calculate some relevant cosmological parameters and their evolution. As a result we find that this model can provide the present cosmic acceleration and even the equation of state parameter of this model can cross the phantom line  $w_D = -1$  provided the model parameters are chosen suitably.

*keywords:* Brans-Dicke; power-law; dark energy.

## I. INTRODUCTION

Recent cosmological and astrophysical data gathered from the observations of SNe Ia [1], WMAP [2], SDSS [3] and X-ray [4] convincingly suggest that the observable universe experiences an accelerated expansion phase. Although the simplest and elegant way to explain this behavior is the inclusion of Einstein's cosmological constant [5], however the two deep theoretical problems

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(namely the “fine-tuning” and the “coincidence” one) led to the dark energy paradigm. The dynamical nature of dark energy, at least in an effective level, can arise from various scalar fields, such as a canonical scalar field (quintessence) [6], a phantom field [7], that is a scalar field with a negative sign of the kinetic term, or the combination of quintessence and phantom in a unified model named quintom [8].

One of the dynamic candidates for dark energy is the so-called “Holographic Dark Energy” (HDE) proposal which is constructed in the light of the holographic principle. Its origin is the black hole thermodynamics [9] and the connection (known from AdS/CFT correspondence) of the UV cut-off of quantum field theory, which gives rise to the vacuum energy, with the largest distance of the theory [10–12]. Thus, determining an appropriate quantity  $L$  to serve as an IR cut-off, imposing the constraint that the total vacuum energy in the corresponding maximum volume must not be greater than the mass of a black hole of the same size, and saturating the inequality, one identifies the acquired vacuum energy as HDE i.e.  $\rho_D = 3c^2 M_P^2 L^{-2}$ . Here  $M_P$  is the reduced Planck Mass  $M_P^{-2} = 8\pi G$ . In this model, the coincidence problem can be resolved by considering a fundamental assumption that matter and HDE do not conserve separately [13]. It was shown [13] that, if there is any interaction between the two dark components of the universe the identification of  $L$  with Hubble radius,  $L = H^{-1}$ , necessarily implies a constant ratio of the energy densities of the two components regardless of the details of the interaction (see also [14]). The HDE model has also been tested and constrained by various astronomical observations where suitable limits have been obtained on the holographic parameter  $c$  and its equation of state parameter [15, 16].

It is worthy to note that the entropy-area relationship  $S(A)$  yields new results in gravitational physics: for instance, the entropy-area relation yields the Friedmann equation and the definition of the HDE. In the later case, the classical relation  $S \sim A \sim L^2$  of black holes yields the dark energy density  $\rho_D = 3c^2 M_P^2 L^{-2}$  [10]. The relation  $S(A)$  has two interesting modifications (corrections) namely the logarithmic correction [17] and power-law correction [18, 23]. In this paper, we are interested in the later case, that is the modification of HDE due to the power-law correction to entropy. These corrections arise in dealing with the entanglement of quantum fields in and out the horizon [18]. The power-law corrected entropy takes the form [23]

$$S = \frac{A}{4G} \left( 1 - K_\alpha A^{1-\alpha/2} \right), \quad (1)$$

where  $\alpha$  is a dimensionless constant whose value is currently under debate, and

$$K_\alpha = \frac{\alpha(4\pi)^{\alpha/2-1}}{(4-\alpha)r_c^{2-\alpha}}, \quad (2)$$

where  $r_c$  is the crossover scale. The second term in (1) is the power-law correction to the entropy-area law. This correction arises when the wave-function of the field is chosen to be a superposition of ground state and excited state [18]. The ground state obeys the usual Bekenstein-Hawking entropy-area law. The corrections to entropy arise only from the excited state, and more excitations produce more deviation from the entropy-area law [20] (also see [21] for a review on power-law entropy corrections).

Motivated by the power-law entropy corrected relation (1), a new version of HDE called “power-law entropy-corrected holographic dark energy” (PLECHDE) was recently proposed [22]

$$\rho_D = 3c^2 M_P^2 L^{-2} - \beta M_P^2 L^{-\alpha}. \quad (3)$$

In the special case  $\beta = 0$ , the above equation yields the well-known HDE density. When  $\alpha = 2$  the two terms can be combined and one recovers again the ordinary HDE density. From thermodynamical point of view, it was shown [23] that the generalized second law of thermodynamics for the universe with the power-law corrected entropy (1) is satisfied provided  $\alpha > 2$ . For  $\alpha > 2$  the corrected term can be comparable to the first term only when  $L$  is very small [22]. Hence, at the very early stage when the Universe undergoes an inflation phase, the correction term in the PLECHDE density (3) becomes important. When the Universe becomes large, PLECHDE reduces to the ordinary HDE. Note that after the end of the inflationary phase, the Universe subsequently enters in the radiation and then matter dominated eras. In these two epochs, since the Universe is much larger, the power-law entropy-corrected term to PLECHDE, namely the second term in Eq. (3), can be safely ignored. Therefore, the PLECHDE can be considered as a model of entropic cosmology which unifies the early-time inflation and late-time cosmic acceleration of the Universe.

Another interesting attempt for probing the nature of dark energy is the so-called “agegraphic dark energy” (ADE). This model was proposed by Cai [24] to explain the acceleration of the universe expansion within the framework of quantum gravity. The ADE model assumes that the observed dark energy comes from the spacetime and matter field fluctuations in the universe. Since the original ADE model suffers from the difficulty to describe the matter-dominated epoch, a new version of ADE was proposed by Wei and Cai [25], while the time scale was chosen to be the conformal time  $\eta$  instead of the age of the universe yielding the new-agegraphic dark energy (NADE) paradigm. The energy density of the NADE is given by [25]

$$\rho_D = \frac{3n^2 M_P^2}{\eta^2}, \quad (4)$$

where the conformal time  $\eta$  is given by

$$\eta = \int_0^a \frac{da}{Ha^2}. \quad (5)$$

The agegraphic models of dark energy have been also investigated in ample details (see e.g [26] and references therein). When the power-law corrections are applied to NADE, the definition modifies to the form [22]

$$\rho_D = 3n^2 M_P^2 \eta^{-2} - \beta M_P^2 \eta^{-\alpha}. \quad (6)$$

On the other hand, HDE/NADE are dynamical dark energy models, thus it is more natural to study them in a dynamical framework such as Brans-Dicke (BD) theory instead of general relativity. Besides, the BD scalar field speeds up the expansion rate of a dust matter dominated era (reduces deceleration), while slows down the expansion rate of cosmological constant era (reduces acceleration). The studies on the HDE and ADE in the framework of BD theory have been carried out in [27, 28] and [29, 30], respectively.

For all mentioned above, it is meaningful to investigate the power-law entropy-corrected HDE/NADE in the framework of BD theory. These studies allows us to show the phantom crossing for the EoS parameter at the present time. We will also study the deceleration parameter and evolutionary form of dark energy density for both interacting and noninteracting cases in the BD framework.

This paper is structured as follows. In the next section we study PLECHDE in BD cosmology. We also calculate the cosmological parameters of the model. In section III we discuss PLECNADE for both interacting and noninteracting cases. The last section is devoted to conclusions.

## II. PLECHDE IN BRANS-DICKE THEORY

The canonical form of the BD action is given by [31]

$$S = \int d^4x \sqrt{g} \left( -\frac{1}{8\omega} \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + L_M \right), \quad (7)$$

where  $R$  and  $\phi$  are the Ricci scalar and the BD scalar field, respectively. Taking the variation of action (7) with respect to the Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (8)$$

yields the Friedmann equations in the framework of BD theory as

$$\frac{3}{4\omega}\phi^2 \left( H^2 + \frac{k}{a^2} \right) - \frac{1}{2}\dot{\phi}^2 + \frac{3}{2\omega}H\dot{\phi}\phi = \rho_D + \rho_M, \quad (9)$$

$$\frac{-1}{4\omega}\phi^2 \left( 2\frac{\dot{a}}{a} + H^2 + \frac{k}{a^2} \right) - \frac{1}{\omega}H\dot{\phi}\phi - \frac{1}{2\omega}\ddot{\phi}\phi - \frac{1}{2}\left(1 + \frac{1}{\omega}\right)\dot{\phi}^2 = p_D, \quad (10)$$

$$\ddot{\phi} + 3H\dot{\phi} - \frac{3}{2\omega}\left(\frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2}\right)\phi = 0. \quad (11)$$

Here,  $\rho_D$  and  $p_D$  are the energy density and pressure of dark energy. Also  $\rho_M$  is the energy density of pressureless dark matter ( $p_M = 0$ ). Following [28], we assume  $\phi \propto a^\varepsilon$ , then one can get

$$\dot{\phi} = \varepsilon H\phi, \quad (12)$$

$$\ddot{\phi} = \varepsilon^2 H^2\phi + \varepsilon\phi\dot{H}. \quad (13)$$

In the framework of BD cosmology, we assume the energy density of the PLECHDE has the following form

$$\rho_D = \frac{3c^2\phi^2}{4\omega L^2} - \frac{\beta\phi^2}{4\omega L^\alpha}, \quad (14)$$

where  $\phi^2 = \omega/(2\pi G_{\text{eff}})$  and  $G_{\text{eff}}$  is the effective gravitational constant. In the limiting case  $G_{\text{eff}} \rightarrow G$ , we have  $\phi^2 = 4\omega M_P^2$  and expression (14) restores the PLECHDE density in Einstein gravity (3). Equation (14) can be rewritten as

$$\rho_D = \frac{3c^2\phi^2}{4\omega L^2}\gamma_c, \quad (15)$$

where

$$\gamma_c = 1 - \frac{\beta}{3c^2L^{\alpha-2}}. \quad (16)$$

The IR cut-off  $L$  is given by [32]

$$L = a(t)\frac{\sin y}{\sqrt{k}}, \quad (17)$$

where  $y = \sqrt{k}R_h/a$  and

$$R_h = a(t) \int_t^\infty \frac{dt}{a(t)} = a(t) \int_0^r \frac{dr}{\sqrt{1-kr^2}}. \quad (18)$$

Here  $R_h$  is the radial size of the event horizon measured in the  $r$  direction and  $L$  is the radius of the event horizon measured on the sphere of the horizon [32].

The critical energy density,  $\rho_{\text{cr}}$ , and the energy density of curvature,  $\rho_k$ , are defined as

$$\rho_{\text{cr}} = \frac{3\phi^2 H^2}{4\omega}, \quad \rho_k = \frac{3k\phi^2}{4\omega a^2}. \quad (19)$$

The dimensionless density parameters can also be defined as usual

$$\Omega_M = \frac{\rho_M}{\rho_{\text{cr}}} = \frac{4\omega\rho_M}{3\phi^2H^2}, \quad (20)$$

$$\Omega_k = \frac{\rho_k}{\rho_{\text{cr}}} = \frac{k}{H^2a^2}, \quad (21)$$

$$\Omega_D = \frac{\rho_D}{\rho_{\text{cr}}} = \frac{c^2\gamma_c}{L^2H^2}. \quad (22)$$

Using Eqs. (12), (20), (21) and (22), one can rewrite the first Friedmann equation (9) as

$$1 + \Omega_k + 2\varepsilon \left(1 - \frac{\varepsilon\omega}{3}\right) = \Omega_D + \Omega_M. \quad (23)$$

From Eq. (22) we have

$$HL = \sqrt{\frac{c^2\gamma_c}{\Omega_D}}. \quad (24)$$

Taking the derivative of Eq. (17) with respect to the cosmic time  $t$  and using (24) yields

$$\dot{L} = \sqrt{\frac{c^2\gamma_c}{\Omega_D}} - \cos y. \quad (25)$$

Taking the time derivative of Eq. (15) and using Eqs. (12) and (25) we get

$$\dot{\rho}_D = H\rho_D \left[ 2\varepsilon + \left(1 - \frac{1}{c}\sqrt{\frac{\Omega_D}{\gamma_c}} \cos y\right) \left(\frac{\alpha-2}{\gamma_c} - \alpha\right) \right]. \quad (26)$$

### A. Noninteracting Case

For the spatially non-flat FRW universe filled with PLECHDE and dark matter, the energy conservation laws are as follows

$$\dot{\rho}_D + 3H\rho_D(1 + w_D) = 0, \quad (27)$$

$$\dot{\rho}_M + 3H\rho_M = 0, \quad (28)$$

where  $w_D = p_D/\rho_D$  is the equation of state (EoS) parameter of PLECHDE. Substituting Eq. (26) in (27), we obtain immediately the EoS parameter of PLECHDE in BD gravity

$$w_D = -1 - \frac{2\varepsilon}{3} - \frac{1}{3\gamma_c} \left(1 - \frac{1}{c}\sqrt{\frac{\Omega_D}{\gamma_c}} \cos y\right) \left((1 - \gamma_c)\alpha - 2\right). \quad (29)$$

For  $\varepsilon = 0$  ( $\omega \rightarrow \infty$ ) the BD scalar field becomes trivial, i.e.  $\phi^2 = \omega/2\pi G = 4\omega M_P^2$ , and Eq. (29) restores the EoS parameter of PLECHDE in Einstein gravity [22]

$$w_D = -1 - \frac{1}{3\gamma_c} \left(1 - \frac{1}{c}\sqrt{\frac{\Omega_D}{\gamma_c}} \cos y\right) \left((1 - \gamma_c)\alpha - 2\right). \quad (30)$$

On the other hand, in the absence of correction term ( $\beta = 0 = \alpha$ ), from Eq. (16) we have  $\gamma_c = 1$  and Eq. (29) reduces to the EoS parameter of HDE in BD gravity [28]

$$w_D = -\frac{1}{3} - \frac{2\varepsilon}{3} - \frac{2\sqrt{\Omega_D}}{3c} \cos y. \quad (31)$$

For the deceleration parameter

$$q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2}, \quad (32)$$

dividing Eq. (10) by  $H^2$ , and using Eqs. (12), (13), (21), (22) and (32), we obtain

$$q = \frac{1}{2\varepsilon + 2} [(2\varepsilon + 1)^2 + 2\varepsilon(\varepsilon\omega - 1) + \Omega_k + 3\Omega_D w_D]. \quad (33)$$

Note that combining the deceleration parameter with the Hubble, the EoS and the dimensionless density parameters, form a set of useful parameters for the description of the astrophysical observations.

Replacing Eq. (29) in (33) yields

$$\begin{aligned} q &= \frac{1}{2\varepsilon + 2} \left[ (2\varepsilon + 1)^2 + 2\varepsilon(\varepsilon\omega - 1) + \Omega_k - (2\varepsilon + 3)\Omega_D - \frac{\Omega_D}{\gamma_c} \right. \\ &\quad \times \left. \left( 1 - \frac{1}{c} \sqrt{\frac{\Omega_D}{\gamma_c}} \cos y \right) ((1 - \gamma_c)\alpha - 2) \right]. \end{aligned} \quad (34)$$

Again for  $\gamma_c = 1$  ( $\alpha = 0$ ) the above equation reduces to [28]

$$q = \frac{1}{2\varepsilon + 2} \left[ (2\varepsilon + 1)^2 + 2\varepsilon(\varepsilon\omega - 1) + \Omega_k - (2\varepsilon + 1)\Omega_D - \frac{2}{c}\Omega_D^{3/2} \cos y \right]. \quad (35)$$

## B. Interacting Case

Here, we extend our investigation to the case in which there is an interaction between PLECHDE and dark matter. The recent observational evidence provided by the galaxy cluster Abell A586 supports the interaction between dark energy and dark matter [33]. In the presence of interaction,  $\rho_D$  and  $\rho_M$  do not conserve separately and the energy conservation equations become

$$\dot{\rho}_D + 3H\rho_D(1 + w_D) = -Q, \quad (36)$$

$$\dot{\rho}_M + 3H\rho_M = Q, \quad (37)$$

where  $Q$  stands for the interaction term. Following [13], we shall assume

$$Q = 3b^2H(\rho_M + \rho_D), \quad (38)$$

with the coupling constant  $b^2$ . Substituting Eqs. (26) and (38) in Eq. (36) and using (23) gives

$$\begin{aligned} w_D = -1 - \frac{2\varepsilon}{3} - \frac{1}{3\gamma_c} \left( 1 - \frac{1}{c} \sqrt{\frac{\Omega_D}{\gamma_c}} \cos y \right) \left( (1 - \gamma_c)\alpha - 2 \right) \\ - \frac{b^2}{\Omega_D} \left[ 1 + \Omega_k + 2\varepsilon \left( 1 - \frac{\varepsilon\omega}{3} \right) \right]. \end{aligned} \quad (39)$$

For  $\gamma_c = 1$  ( $\alpha = 0$ ), the above equation reduces to the EoS parameter of interacting HDE in BD cosmology [28]

$$w_D = -\frac{1}{3} - \frac{2\varepsilon}{3} - \frac{2\sqrt{\Omega_D}}{3c} \cos y - \frac{b^2}{\Omega_D} \left[ 1 + \Omega_k + 2\varepsilon \left( 1 - \frac{\varepsilon\omega}{3} \right) \right]. \quad (40)$$

In the presence of interaction, the deceleration parameter for PLECNADE model can be obtained by replacing Eq. (39) in (33) as

$$\begin{aligned} q = \frac{1}{2\varepsilon + 2} \left[ (2\varepsilon + 1)^2 + 2\varepsilon(\varepsilon\omega - 1) + \Omega_k - (2\varepsilon + 3)\Omega_D - \frac{\Omega_D}{\gamma_c} \right. \\ \times \left( 1 - \frac{1}{c} \sqrt{\frac{\Omega_D}{\gamma_c}} \cos y \right) \left( (1 - \gamma_c)\alpha - 2 \right) - 3b^2 \left[ 1 + \Omega_k + 2\varepsilon \left( 1 - \frac{\varepsilon\omega}{3} \right) \right] \left. \right]. \end{aligned} \quad (41)$$

Again for  $\gamma_c = 1$  ( $\alpha = 0$ ) we have [28]

$$\begin{aligned} q = \frac{1}{2\varepsilon + 2} \left\{ (2\varepsilon + 1)^2 + 2\varepsilon(\varepsilon\omega - 1) + \Omega_k - (2\varepsilon + 1)\Omega_D - \frac{2}{c}\Omega_D^{3/2} \cos y \right. \\ \left. - 3b^2 \left[ 1 + \Omega_k + 2\varepsilon \left( 1 - \frac{\varepsilon\omega}{3} \right) \right] \right\}. \end{aligned} \quad (42)$$

Taking time derivative of Eq. (24) and using  $\dot{\Omega}_D = H\Omega'_D$ , one can get the equation of motion for  $\Omega_D$  as

$$\Omega'_D = \Omega_D \left[ 2 + 2q + \left( \frac{\alpha - 2}{\gamma_c} - \alpha \right) \left( 1 - \frac{1}{c} \sqrt{\frac{\Omega_D}{\gamma_c}} \cos y \right) \right], \quad (43)$$

where the prime denotes the derivative with respect to  $x = \ln a$  and  $q$  is given by Eq. (41). In the absence of correction we have  $\gamma_c = 1$  ( $\beta = 0 = \alpha$ ), thus Eq. (43) restores [28]

$$\Omega'_D = 2\Omega_D \left( q + \frac{\sqrt{\Omega_D}}{c} \cos y \right). \quad (44)$$

### III. PLECNADE IN BRANS-DICKE THEORY

The PLECNADE density in BD gravity is given by

$$\rho_D = \frac{3n^2\phi^2}{4\omega\eta^2} - \frac{\beta\phi^2}{4\omega\eta^\alpha}. \quad (45)$$

We rewrite Eq. (45) as

$$\rho_D = \frac{3n^2\phi^2}{4\omega\eta^2}\gamma_n, \quad (46)$$

with

$$\gamma_n = 1 - \frac{\beta}{3n^2\eta^{\alpha-2}}. \quad (47)$$

From definition  $\rho_D = \Omega_D\rho_{\text{cr}} = 3\phi^2H^2\Omega_D/(4\omega)$  and using Eq. (46), we get

$$H\eta = \sqrt{\frac{n^2\gamma_n}{\Omega_D}}. \quad (48)$$

Taking time derivative of Eq. (46), using (12) and  $\dot{\eta} = 1/a$  we obtain

$$\dot{\rho}_D = H\rho_D \left[ 2\varepsilon + \frac{1}{an} \sqrt{\frac{\Omega_D}{\gamma_n}} \left( \frac{\alpha-2}{\gamma_n} - \alpha \right) \right]. \quad (49)$$

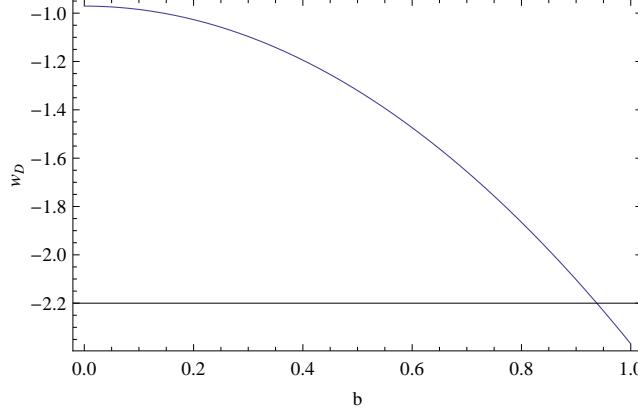


FIG. 1: The equation of state parameter of PLECNADE is plotted against interacting coupling parameter  $b$ . The trial values of model parameters are  $\omega = 10^4$ ,  $\omega\epsilon \approx 1$ ,  $\alpha = 2.8$ ,  $\Omega_k = 0.02$ ,  $n = 2.716$ ,  $a = 1$ ,  $\gamma_n = 100$ ,  $\Omega_D = 0.73$ .

### A. Noninteracting Case

For noninteracting PLECNADE in BD theory, the EoS parameter can be obtained by replacing Eq. (49) in (27). The result is

$$w_D = -1 - \frac{2\varepsilon}{3} - \frac{1}{3an} \sqrt{\frac{\Omega_D}{\gamma_n}} \left( \frac{\alpha-2}{\gamma_n} - \alpha \right). \quad (50)$$

In the special case  $\gamma_n = 1$  ( $\alpha = 0$ ), Eq. (50) restores the EoS parameter of NADE in BD gravity [29]

$$w_D = -1 - \frac{2\varepsilon}{3} + \frac{2\sqrt{\Omega_D}}{3na}. \quad (51)$$

The deceleration parameter  $q$  for the noninteracting PLECNADE model is still obtained according to Eq. (33), where  $w_D$  is now given by Eq. (50). Substituting Eq. (50) in (33) gives

$$\begin{aligned} q = & \frac{1}{2\varepsilon + 2} \left[ (2\varepsilon + 1)^2 + 2\varepsilon(\varepsilon\omega - 1) + \Omega_k - (2\varepsilon + 3)\Omega_D - \frac{\Omega_D^{3/2}}{an\sqrt{\gamma_n}} \right. \\ & \times \left. \left( \frac{\alpha - 2}{\gamma_n} - \alpha \right) \right]. \end{aligned} \quad (52)$$

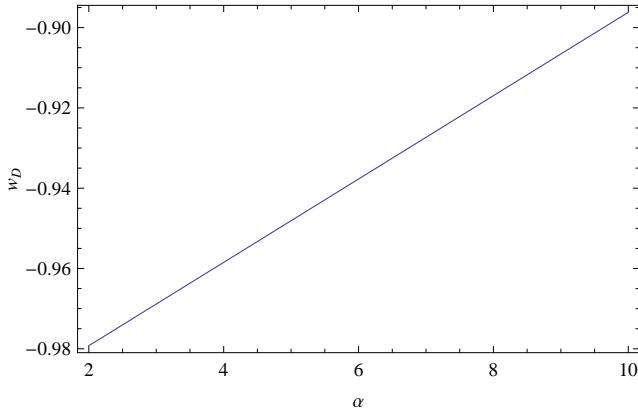


FIG. 2: The equation of state parameter of PLECNADE (Eq.53) is plotted against power-law parameter  $\alpha$ . The trial values of model parameters are  $\omega = 10^4$ ,  $\omega\epsilon \approx 1$ ,  $\Omega_k = 0.02$ ,  $n = 2.716$ ,  $a = 1$ ,  $b = 0.01$ ,  $\gamma_n = 100$ ,  $\Omega_D = 0.73$ .

### B. Interacting Case

Here, the EoS parameter of interacting PLECNADE in BD gravity is obtained by replacing Eqs. (38) and (49) into Eq. (36) and using (23). The result yields

$$w_D = -1 - \frac{2\varepsilon}{3} - \frac{1}{3an} \sqrt{\frac{\Omega_D}{\gamma_n}} \left( \frac{\alpha - 2}{\gamma_n} - \alpha \right) - \frac{b^2}{\Omega_D} \left[ 1 + \Omega_k + 2\varepsilon \left( 1 - \frac{\varepsilon\omega}{3} \right) \right]. \quad (53)$$

For  $\gamma_n = 1$  ( $\beta = 0 = \alpha$ ), the above equation reduces to [29]

$$w_D = -1 - \frac{2\varepsilon}{3} + \frac{2\sqrt{\Omega_D}}{3na} - \frac{b^2}{\Omega_D} \left[ 1 + \Omega_k + 2\varepsilon \left( 1 - \frac{\varepsilon\omega}{3} \right) \right]. \quad (54)$$

The deceleration parameter is obtained by replacing Eq. (53) in (33)

$$q = \frac{1}{2\varepsilon + 2} \left\{ (2\varepsilon + 1)^2 + 2\varepsilon(\varepsilon\omega - 1) + \Omega_k - (2\varepsilon + 3)\Omega_D - \frac{\Omega_D^{3/2}}{an\sqrt{\gamma_n}} \right. \\ \left. \times \left( \frac{\alpha - 2}{\gamma_n} - \alpha \right) - 3b^2 \left[ 1 + \Omega_k + 2\varepsilon \left( 1 - \frac{\varepsilon\omega}{3} \right) \right] \right\}. \quad (55)$$

Taking time derivative of Eq. (48), using  $\dot{\Omega}_D = H\Omega'_D$  and  $\dot{\eta} = 1/a$ , the equation of motion for  $\Omega_D$  can be obtained as

$$\Omega'_D = \Omega_D \left[ 2 + 2q + \frac{1}{an} \sqrt{\frac{\Omega_D}{\gamma_n}} \left( \frac{\alpha - 2}{\gamma_n} - \alpha \right) \right], \quad (56)$$

which in the absence of correction term  $\gamma_n = 1$  ( $\alpha = 0$ ) reduces to the result obtained for NADE in BD gravity [29]

$$\Omega'_D = 2\Omega_D \left[ 1 + q - \frac{1}{an} \sqrt{\Omega_D} \right]. \quad (57)$$

In figures 1 and 2, we have plotted the equation of state parameter of PLECNADE against various model parameters. From these figures one can see explicitly that the equation of state parameter can cross the phantom boundary  $w_D = -1$ , thus realizing the phenomenon of cosmic acceleration.

#### IV. CONCLUSIONS

In this paper, we investigated the models of HDE and NADE with power-law correction taking a non-flat FRW background in the BD gravitational theory. The power-law correction is motivated from the entanglement of quantum fields in and out the horizon. The BD theory of gravity involves a scalar field which accounts for a dynamical gravitational constant. We assumed an ansatz by which the BD scalar field evolves with the expansion of the universe. We then established a correspondence between the field and the PLECHDE (and PLECNADE) to study its dynamics. The dynamics are governed by few dynamical parameters like its equation of state, deceleration parameter and energy density parameter. For the sake of generality, we calculated them in the non-flat background with the interaction of PLECHDE (and PLECNADE) with the matter. Interestingly enough we found that the presented model can accomodate the phantom regim for the equation of state parameter provided the model parameters are chosen suitably. To clarify this point we plotted the evolution of  $w_D$  against scale factor and demonstrated explicitly that cosmic acceleration and phantom crossing can be realized in our model.

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